CORRECTIONS TO "FAST FINITE-ENERGY PLANES IN SYMPLECTIZATIONS AND APPLICATIONS"

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Abstract. We correct the proof of Lemma 6.24.

In the proof below we use the same equation numbers and reference numbers as in the published version.

Proof of Lemma 6.24. It follows from $\text{wind}_\pi(\tilde{u}) = \text{wind}_\infty(\tilde{u}) - 1 = 0$ that $\pi \cdot du$ does not vanish. Hence u and x only intersect transversely. Let us assume, by contradiction, that $u(\mathbb{C}) \cap x(\mathbb{R}) \neq \emptyset$. In view of Definition 3.11, the map u has self-intersections and we find intersections of the planes $\tilde{u}$ and $c \cdot \tilde{u}$ for $c \gg 1$. Here $c \cdot \tilde{u}$ denotes the translation of the plane $\tilde{u}$ in the $\mathbb{R}$-direction of $\mathbb{R} \times M$ by c, as explained in Remark 3.10. Consider, for each $c > 0$, the closed set

$$A_c = \{(z, w) \in \mathbb{C} \times \mathbb{C} : \tilde{u}(z) = c \cdot \tilde{u}(w)\}.$$

A_c can not accumulate in $\mathbb{C} \times \mathbb{C}$ since otherwise one could argue, using the similarity principle as in [26], that $\tilde{u}(\mathbb{C}) = (c \cdot \tilde{u})(\mathbb{C})$. There exists $R_0 \gg 1$ such that

$$R \geq R_0 \Rightarrow \begin{cases} \tilde{u}^{-1}\left(\tilde{u}\left(\mathbb{C} \setminus B_R(0)\right)\right) = \mathbb{C} \setminus B_R(0) \\ u|_{\mathbb{C} \setminus B_R(0)} : \mathbb{C} \setminus B_R(0) \to M \setminus x(\mathbb{R}) \text{ is an embedding.} \end{cases} \tag{100}$$

This follows from the facts that $\tilde{u}$ is a proper embedding, and that its asymptotic limit $P = (x, T)$ is simply covered.

Fix $c_0 > 0$ such that $A_{c_0} \neq \emptyset$. It follows from (100) that for every $\epsilon \in (0, c_0]$ there exists a compact $K \subset \mathbb{C}$ such that $A_c \subset K \times K$ holds for every $c \in [\epsilon, c_0]$. To prove this we argue by contradiction. If there is no such K for some given $\epsilon \in (0, c_0]$ then we find a sequence $b_n \in [\epsilon, c_0]$ and a sequence $(z_n, w_n) \in \mathbb{C} \times \mathbb{C}$ with no accumulation points such that $u(z_n) = u(w_n)$, $a(z_n) = a(w_n) + b_n$ for all n. Then it must be true that $z_n \neq w_n$ for all n. Up to selection of a subsequence we may assume that the limits $\lim_{n\to\infty} |z_n|$ and $\lim_{n\to\infty} |w_n|$ exist in $[0, +\infty]$, and that at least one of these limits is equal to $+\infty$. If $\lim_{n\to\infty} |z_n| = \lim_{n\to\infty} |w_n| = +\infty$ then we get a direct contradiction to (100). Assume that $\lim_{n\to\infty} |z_n| < +\infty$ in which case we must necessarily have $|w_n| \to +\infty$. Up to choice of a further subsequence, we may assume $z_n \to z_*$ for some $z_* \in \mathbb{C}$. But since $a(w_n) \to +\infty$ and $\{b_n\}$ is bounded, the identity $a(z_n) = a(w_n) + b_n$ forces $a(z_n) \to +\infty$, in contradiction to $a(z_n) \to a(z_*)$. The case where $\lim_{n\to\infty} |w_n| < +\infty$ is analogous.

We can now use the homotopy invariance of intersection numbers together with positivity of intersections of pseudo-holomorphic maps to conclude that $\tilde{u}$ intersects $c \cdot \tilde{u}$ for every $c \in (0, c_0]$. Let

$$f : \mathbb{C} \times B_r(0) \to \mathbb{R} \times M$$

be the embedding (91) obtained by the implicit function theorem. Choose $c_n \to 0^+$. By Lemma 6.20 there exists $\tau_n \to 0$ satisfying $\tau_n \neq 0$ and $(c_n \cdot \tilde{u})(\mathbb{C}) = f(\mathbb{C}, \tau_n)$ for n large enough. This is an absurd because f is 1-1 and $c_n \cdot \tilde{u}$ intersects $\tilde{u}$ for every n. $\square$

References

[26] D. McDuff and D. Salamon. J-holomorphic curves and symplectic topology. Amer. Math. Soc. Colloq. Publ. 52 (2004).