

## PROSEMINAR ZUR ANALYSIS

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### General information:

Our seminar will be focused on Fourier series, and on its applications to problems in abstract Mathematics. The choice of applications is biased, with emphasis on differential equations coming from Mathematical Physics.

### Topics:

- Fourier Series and their convergence properties.
- Poincaré's rotation number, and dynamical systems on the circle.
- Basic Ergodic Theory and applications to Number Theory.
- Fourier series for the action functional from Classical Mechanics.
- Fourier series for the wave and heat equations. Applications to the physics of sound and music.

### Organisation:

- Introductory lectures: Fourier series and basic convergence properties, overview of the applications.
- Participants will continuously present on the selected topics.
- Distribution of topics:
  - (a) Rotation numbers of circle homeomorphisms. ([1, Chap. III, § 11])
  - (b) Analytic circle homeomorphisms with Diophantine rotation numbers. ([1, Chap. III, § 12])
  - (c) Ergodic theory for linear systems on the torus, and applications to number theory. ([2, Chap. 10, §§ 51, 52])
  - (d) Calculus of variations for the action functional via Fourier series. ([7, Chap. 3])
  - (e) Fourier series for the wave and the heat equations with periodic boundary conditions, in one or many dimensions. ([5, 6])
  - (f) Applications to the physics of sound and music. ([3])
- Evaluation is based on the presentations, and on a written report (10-15 pages).

### REFERENCES

- [1] V. I. Arnol'd, *Geometrical methods in the theory of ordinary differential equations*, 2nd ed., Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 250, Springer-Verlag, New York, 1988.
- [2] ———, *Mathematical methods of classical mechanics*, Graduate Texts in Mathematics, vol. 60, Springer-Verlag, New York, [1989?].
- [3] D. Benson, *Music: a mathematical offering*, Cambridge University Press, Cambridge, 2007.

- [4] R. Courant, *Differential and integral calculus. Vol. I*, Wiley Classics Library, John Wiley & Sons, Inc., New York, 1988.
- [5] R. Courant and D. Hilbert, *Methods of mathematical physics. Vol. I*, Interscience Publishers, Inc., New York, N.Y., 1953.
- [6] L. C. Evans, *Partial differential equations*, 2nd ed., Graduate Studies in Mathematics, vol. 19, American Mathematical Society, Providence, RI, 2010.
- [7] H. Hofer and E. Zehnder, *Symplectic invariants and Hamiltonian dynamics*, Modern Birkhäuser Classics, Birkhäuser Verlag, Basel, 2011.