

Introduction:

Symplectic linear algebra.

Talk 1 (N. Nguyen): symplectic vector spaces, symplectic basis, definition of $Sp(2n)$, basic spectral properties of matrices in $Sp(2n)$ ($\det = 1$), subspaces: isotropic, coisotropic and lagrangian, completion of basis, examples: graph of symplectic linear map etc.

Talk 2 (D. Sinkin): relations between $Sp(2n)$, $O(2n)$, $GL(n, \mathbb{C})$ and $U(n)$, contracting $Sp(2n)$ onto $U(n)$, connectedness of $Sp(2n)$, powers of positive symmetric symplectic matrices are symplectic, space of compatible complex structures as $Sp(2n)/U(n)$ (contractibility).

Talk 3 (A. Lagemann): Maslov index for loops in $Sp(2n)$, the fundamental group of $Sp(2n)$, Maslov index for loops of lagrangian subspaces.

Symplectic manifolds.

Talk 1 (P. Mork): definition of symplectic manifold, symplectic form is not exact on a closed manifold, examples (cotangent bundle, torus, $\mathbb{C}P^n$), S^{2n} can not be symplectic if $n \geq 2$, Darboux's theorem for symplectic forms (Moser's trick).

Talk 2: symplectic vector fields and symplectomorphisms, Hamiltonian vector fields and Hamiltonian diffeomorphisms, Poisson bracket, examples: symplectomorphisms that are not Hamiltonian diffeomorphisms, phase space of the pendulum, Kepler problem (angular momentum is an integral).

Talk 3: lagrangian submanifolds, Weinstein's lagrangian tubular neighbourhood theorem, Arnold conjecture for Hamiltonian diffeomorphisms C^1 -near the identity.

Projects:

Project 1: Arnold-Liouville theorem

- Talk 1: statement of the theorem, example in one degree of freedom: action-angle coordinates for the pendulum, example in two degrees of freedom: Kepler's problem.
- Talk 2: Proof of the theorem.

Project 2: Symplectic capacities and Gromov's non-squeezing theorem

- Talk 1: symplectic capacities, Gromov's non-squeezing theorem as a consequence, Eliashberg-Gromov rigidity as a consequence.
- Talk 2: Definition of the Hofer-Zehnder capacity, selected details of the existence proof.

Project 3: Knots and links in contact 3-manifolds

- Talk 1: contact forms, contact structures, examples (unit cotangent bundles, 1-jet spaces, standard spheres), Reeb flows, geodesic flows as Reeb flows, legendrian submanifolds and examples.
- Talk 2: Darboux's theorem for contact forms, legendrian tubular neighbourhood theorem, self-linking number, Thurston-Bennequin invariant, rotation number, Thurston-Bennequin inequality.

Project 4: Group of Hamiltonian diffeomorphisms and Hofer geometry

- Talk 1: Space of normalized Hamiltonians \mathcal{A} , The group of Hamiltonian diffeomorphisms on M as a Lie group (Lie Algebra, Adjoint action, Lie bracket), Hofer's metric as a variational problem on paths of Ham. diffeomorphisms, choosing L^p -norm vs L^∞ -norm on \mathcal{A} , displacement energy e , upper bounds of e , Hofer's metric non-degenerate if and only if $e(U) > 0$ for all open U .
- Talk 2: non-degeneracy of Hofer's metric. selected details of the proof if $M = \mathbb{R}^n$. (possible proofs: via inequality of displacement energy and Hofer-Zehnder capacity, via inequality with Liouville class of closed rational Lagrangians), diameter and fundamental group if M is a closed surface.